

For the midterm, you should be comfortable with the following terms and theorems and be able to apply the following computational techniques:

**Definitions:**

- Inner product
- Norm, distance, angle
- Orthogonal transformation
- Determinant
- Cofactor
- Adjoint
- Eigenvalue
- Eigenvector
- Eigenspace
- Invariant subspace
- Direct sum
- Minimal polynomial
- Characteristic polynomial
- Companion matrix

**Theorems:**

- Cauchy-Schwartz inequality:  $|\langle \vec{u}, \vec{v} \rangle| \leq \|\vec{u}\| \|\vec{v}\|$
- Triangle inequality:  $\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$
- Uniqueness of determinants
- $D(AB) = D(A)D(B)$
- $A^{-1} = D(A)^{-1} \text{Adj}(A)$
- Primary decomposition
- Triangular form
- Rational canonical form

**Computational Techniques:**

- Compute determinants using row operations
- Compute determinants using row/column expansion
- Compute determinants using complete expansion
- Find eigenvalues/vectors/spaces
- Diagonalize a matrix
- Write a matrix in block-diagonal form using the primary decomposition
- Find rational canonical form given a list of elementary divisors