

For the midterm, you should be comfortable with the following terms and theorems and be able to apply the following computational techniques:

**Definitions:**

- Field (and axioms)
- $\mathbb{Z}_n$
- Zero divisor
- Vector space (and axioms)
- Subspace
- Linear Combination
- Span
- Linear Independence
- Basis
- Dimension
- Row Space
- Column Space
- Rank of a matrix
- Linear Transformation

**Theorems:**

- The intersection of two subspaces is a subspace
- A set of more than  $m$  vectors in a space spanned by  $m$  vectors is linearly dependent
- Every basis of a vector space  $V$  has the same number of elements
- Every spanning set contains a basis
- Every linearly independent set can be completed to a basis
- $\dim(S + T) + \dim(S \cap T) = \dim(S) + \dim(T)$
- The rows of a matrix in echelon form are linearly independent
- If  $A \in M_{m,n}(\mathbb{F})$  then  $\dim(\text{Im}(A)) + \dim(\text{Ker}(A)) = n$ .

**Computational Techniques:**

- Do arithmetic in  $\mathbb{Z}_n$
- Prove theorems by induction
- Find a basis for the row space of a matrix
- Find a basis for the span of a set of vectors in  $\mathbb{F}^n$
- Find a basis for the null space of a matrix
- Row-reduce a matrix to reduced echelon form using Gauss-Jordan elimination
- Find all solutions to a homogeneous or nonhomogeneous system
- Find the matrix of a composition of linear transformations
- Multiply matrices