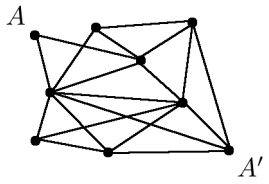
Motivation

Consider the following problems:

1. Your department produces three products which we'll call α, β and γ. Each unit of α generates \$50 in revenue and requires \$10 in labor and \$5 in materials to produce; each unit of β generates \$55 in revenue and requires \$15 in labor and \$5 in materials, and each unit of γ generates \$60 in revenue and requires \$15 in labor and \$10 in materials to produce. For a given production cycle you have a materials budget of \$10,000, a labor budget of \$20,000 and must generate \$79,000 in revenue. How many units each of α, β and γ should your department make in order to meet the required numbers for revenue, labor and materials?

2. A network of airports (or subway stops, or computers, etc.) is represented by the picture below.



How many routes with no more than 4 steps will get a traveler (or packet, etc.) from point A to point A'? How can you be sure you've found them all?

3. City planners need to predict the demand for housing in various districts as people move from one district to another. For instance, a randomly chosen person in district 1 has a 15% probability of moving to district 2, a 10% probability of moving to district 3, and a 75% probability of staying put in district 1. Given today's distribution of population, predict the population distribution in 10 years.

The mathematics required to solve these problems is known as *linear algebra*, the subject of this course. In addition to such practical down-to-earth problems, linear algebra also shows us how to solve many much more abstract problems such as:

4. Higher-dimensional geometry. Two planes intersect in 7-dimensional space; what angle do they make? How can we compute the distance between points in 6-dimensional space, or draw 2-d (or 3-d) shadows of 4-d objects?

5. Differential equations. Suppose $y(x)$, $u(x)$, $v(x)$ and $w(x)$ are unknown functions of x satisfying

$$\begin{aligned} 2y' + 3u' - v' + w' &= x^2 \\ y' - u' + 2v' + w' &= 2x \end{aligned}$$

Find not just one, but all possible solutions.

6. Abstract algebra. What, really, is $\sqrt{-1}$? Are there other types of numbers beyond the complex numbers? How can we deal with, for example, non-commutative multiplication?