Math 60 **Name** _____

Homework 1 **Due Monday September 12 before 4:00 PM**

1. An element in a number system is called a *unit* if it has a multiplicative inverse. Identify the units of $\mathbb{Z}_9$ by finding their multiplicative inverses. For each non-unit element, show that no element of $\mathbb{Z}_9$ works as a multiplicative inverse.

2. A nonzero element of a number system $\beta \neq 0$ is a *zero divisor* if there is a nonzero element $\alpha \neq 0$ such that $\alpha\beta = 0$. Prove that if $\mathbb{F}$ is a field, then $\mathbb{F}$ contains no zero divisors.

3. Let $\mathbb{F}$ be a field and $\alpha, \beta, \gamma \in \mathbb{F}$. Using the field axioms, show that if $\alpha\beta = \alpha\gamma$ and $\alpha \neq 0$, then $\beta = \gamma$.

4. Prove by induction that $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.