Math 60 **Final Exam Review**

For the midterm, you should be comfortable with the following terms and theorems and be able to apply the following computational techniques.

Defintions:

- Field axioms
- Modular Arithmetic
- Vector space axioms
- Subspaces
- Linear Combinations
- Linear Independence
- Basis
- Dimension & Rank
- Row & Column Spaces
- Linear Transformations
- Inner product
- Norm, distance, angle
- Orthogonal transformation
- Change of Basis matrix
- Determinant
- Cofactor
- Adjoint
- Eigenvalue/vector/space
- Invariant subspace
- Direct sum
- Minimal & Characteristic polynomials
- Primary Decomposition
- Dual vectors
- Tensor product
- Ajdacency matrix
- Stochastic matrix

Theorems:

- The intersection of two subspaces is a subspace
- A set of more than m vectors in a space spanned by m vectors is linearly dependent
- Every spanning set contains a basis
- Every linearly independent set can be completed to a basis
- $\dim(S+T) + \dim(S \cap T) = \dim(S) + \dim(T)$
- The rows of a matrix in echelon form are linearly independent
- If $A \in M_{m,n}(\mathbb{F})$ then $\dim(\text{Im}(A)) + \dim(\text{Ker}(A)) = n$.
- Cauchy-Schwartz inequality: $|\langle \vec{u}, \vec{v} \rangle| \leq ||\vec{u}||\ ||\vec{v}||$
- Triangle inequality: $||\vec{u} + \vec{v}|| \leq ||\vec{u}|| + ||\vec{v}||$
- Uniqueness of determinants
- $D(AB) = D(A)D(B)$
- $A^{-1} = D(A)^{-1}\text{Adj}(A)$

Computational Techniques:

- Prove theorems by induction
- Find a basis for the span of a set of vectors in $\mathbb{F}^n$
- Find a basis for the null space of a matrix
- Row-reduce a matrix with entries in $\mathbb{R}$ or $\mathbb{Z}_p$ to reduced echelon form using Gauss-Jordan elimination
- Find all solutions to a homogeneous or nonhomogeneous system

- Find the sum, scalar product, product, direct sum and tensor product of matrices
- Compute determinants using row operations, row/column expansion and complete expansion
- Find eigenvalues/vectors/spaces using the characteristic polynomial
- Find the minimal polynomial of a vector with respect to a linear transformation
- Write a matrix in block-diagonal form using the primary decomposition
- Find rational canonical form given a list of elementary divisors
- Count paths in a graph using powers of the adjacency matrix
- Find stable states in a stochastic system
- Encrypt/decrypt texts using an invertible matrix